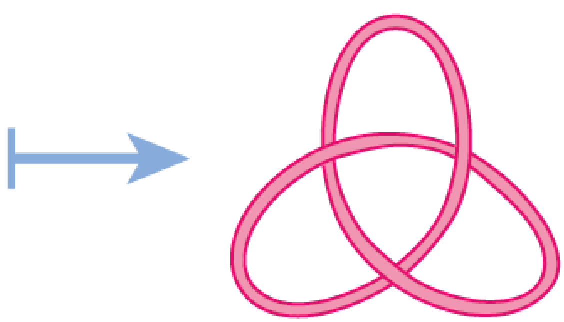
結び目の行列表示とパズルについて

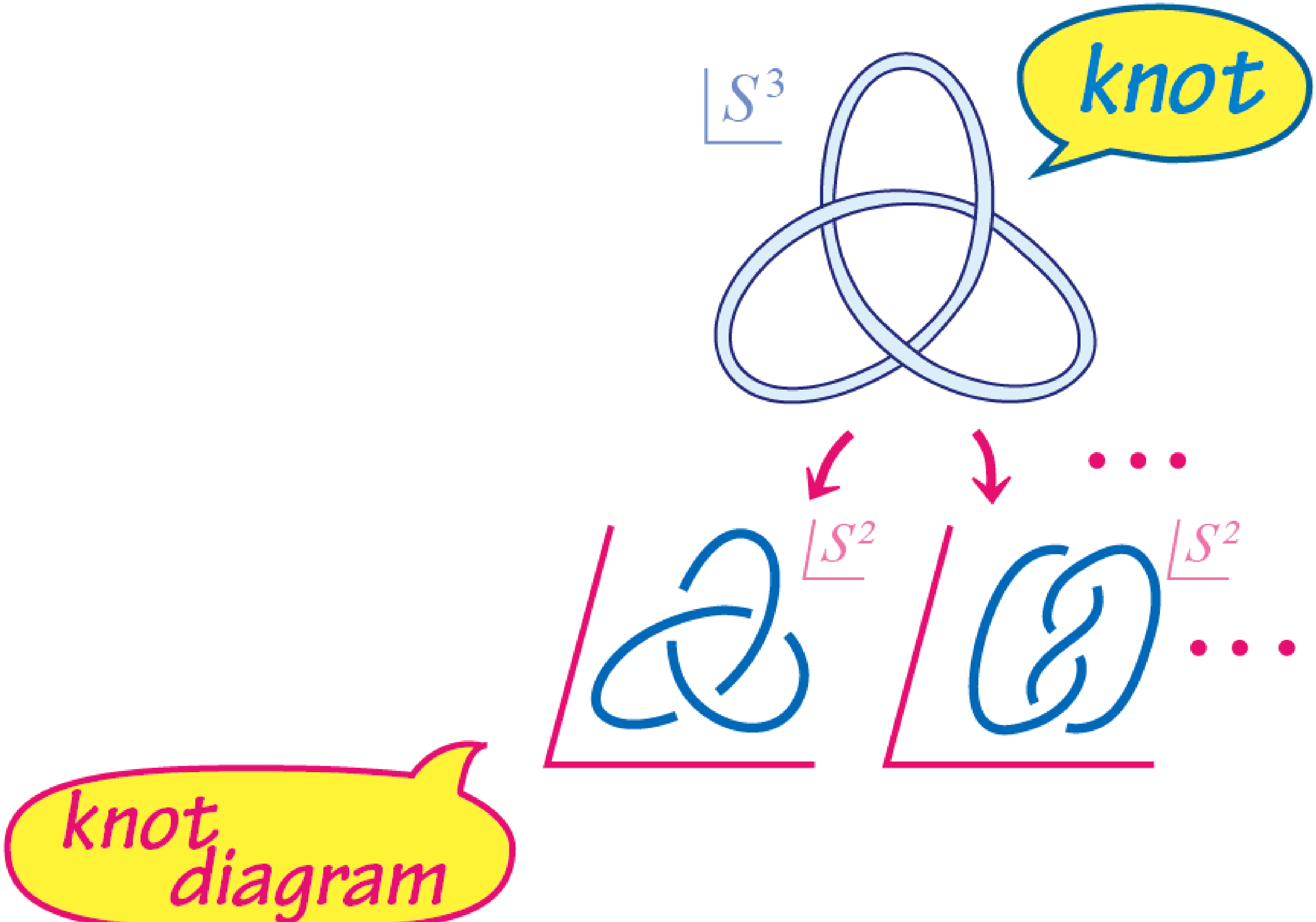
				3	
		3			
				0	
3			0		
	1				1
	2				2
0			3		
		0			



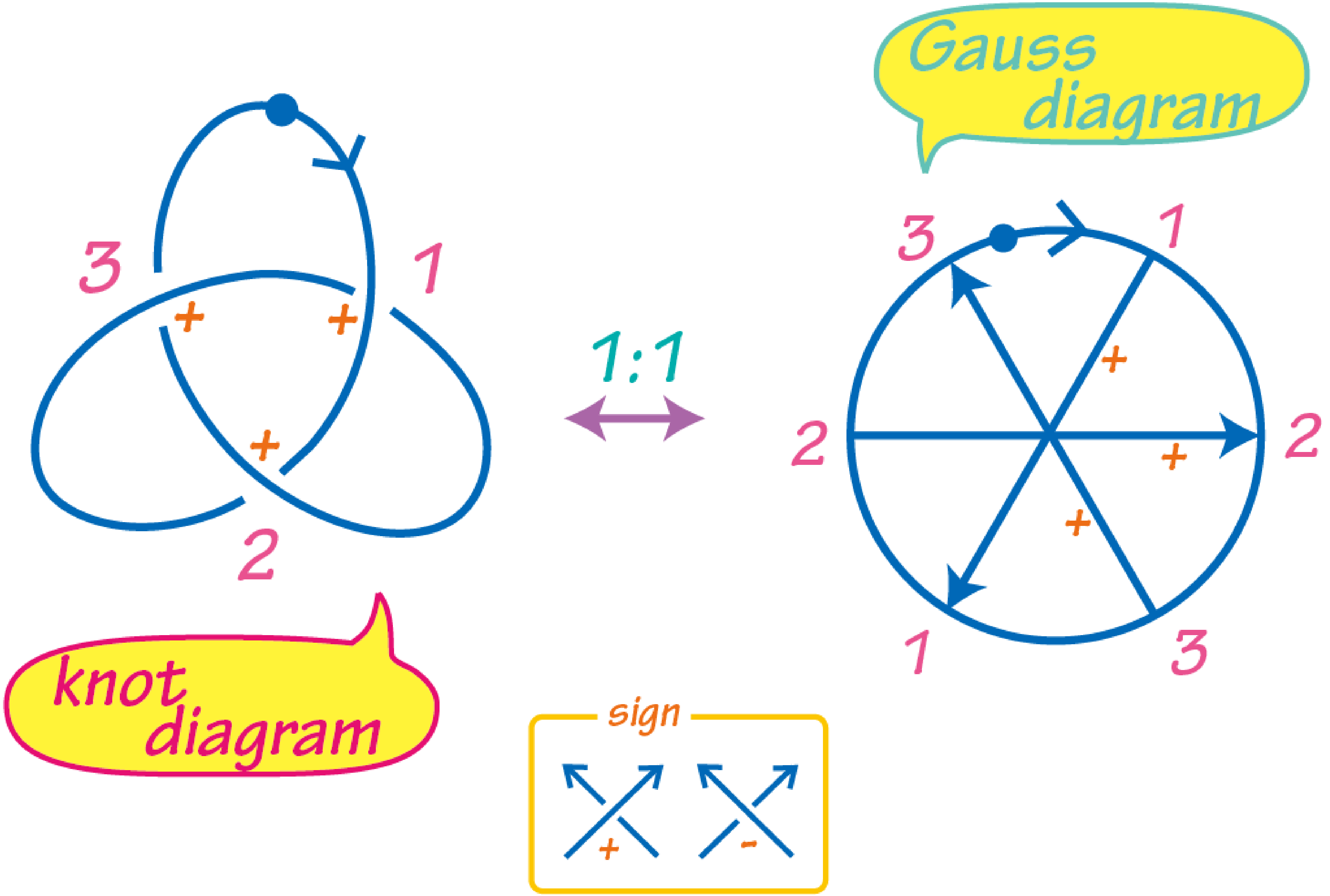
by 清水 理佳

CONTENTS

1. Warping degree



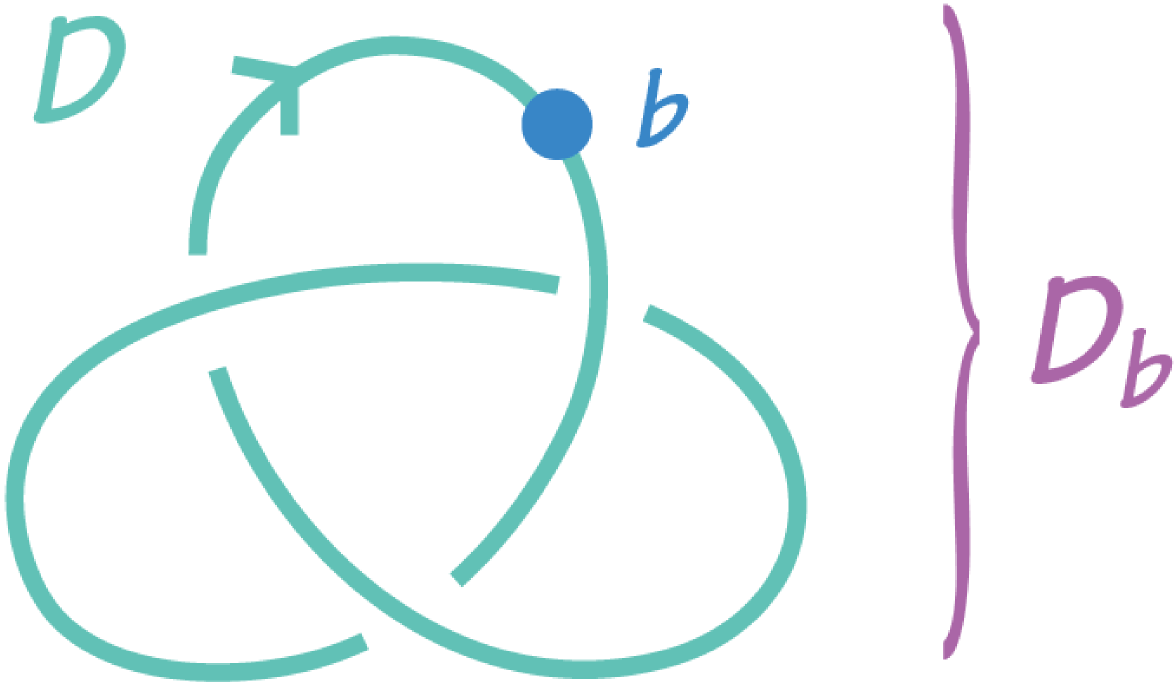
S^3
knot
S^2
S^2
knot diagram



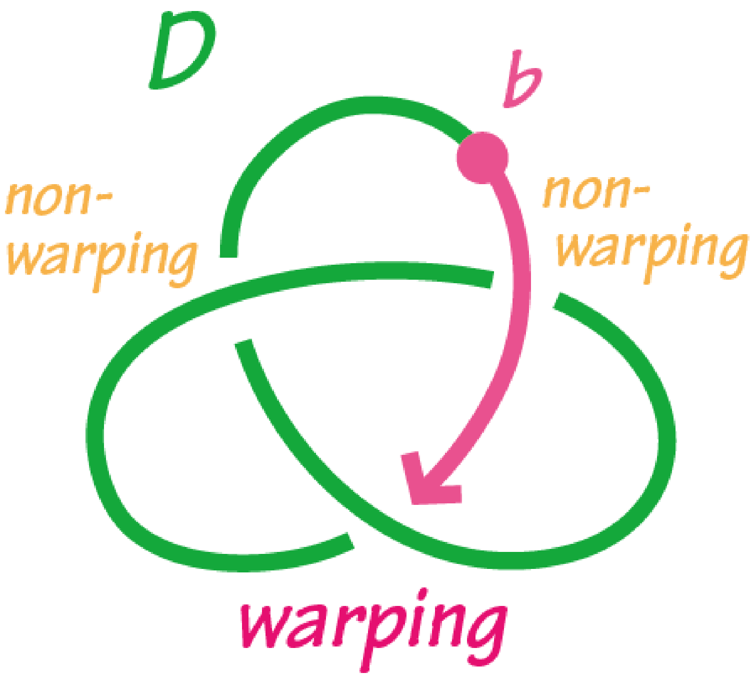
Gauss diagram
knot diagram
1:1
sign
3
1
2
+
-

D: an oriented knot diagram on S^2

b: a base point of D



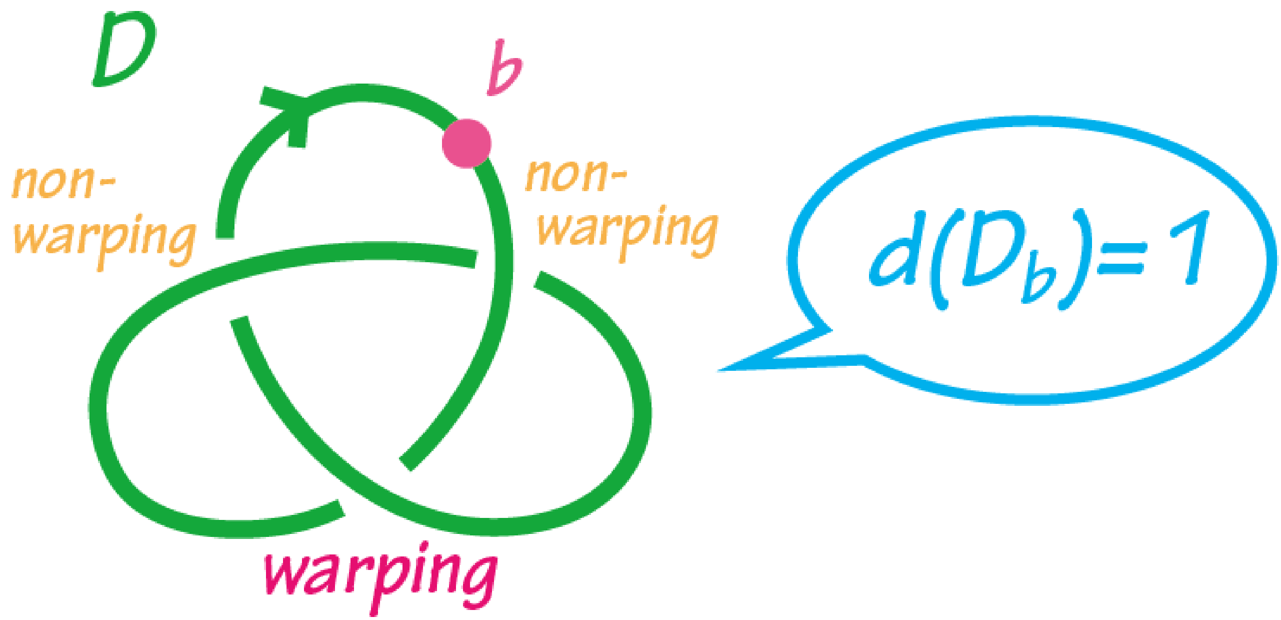
p: a crossing point of D



p is a **warping crossing point** of D_b if we meet p as an undercrossing first when we travel D from b.

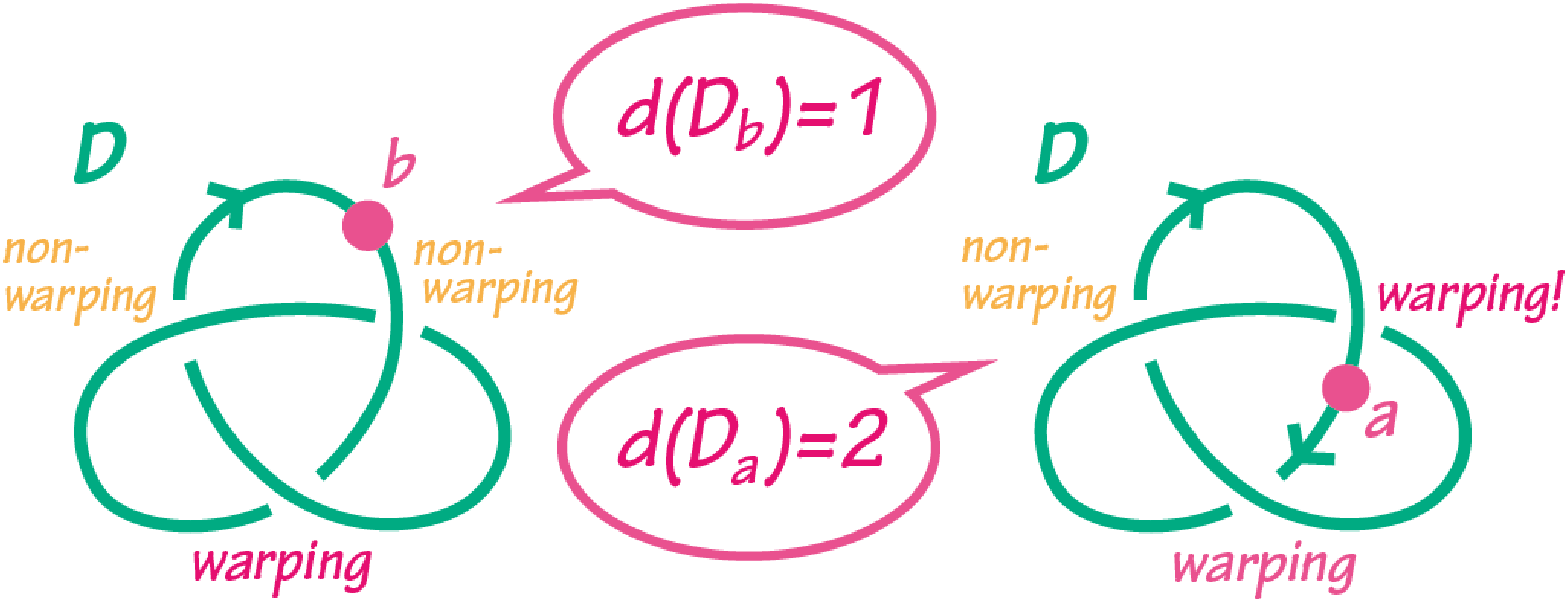
The warping degree $d(D_b)$ of D_b is the number of warping crossing points of D_b.

Example

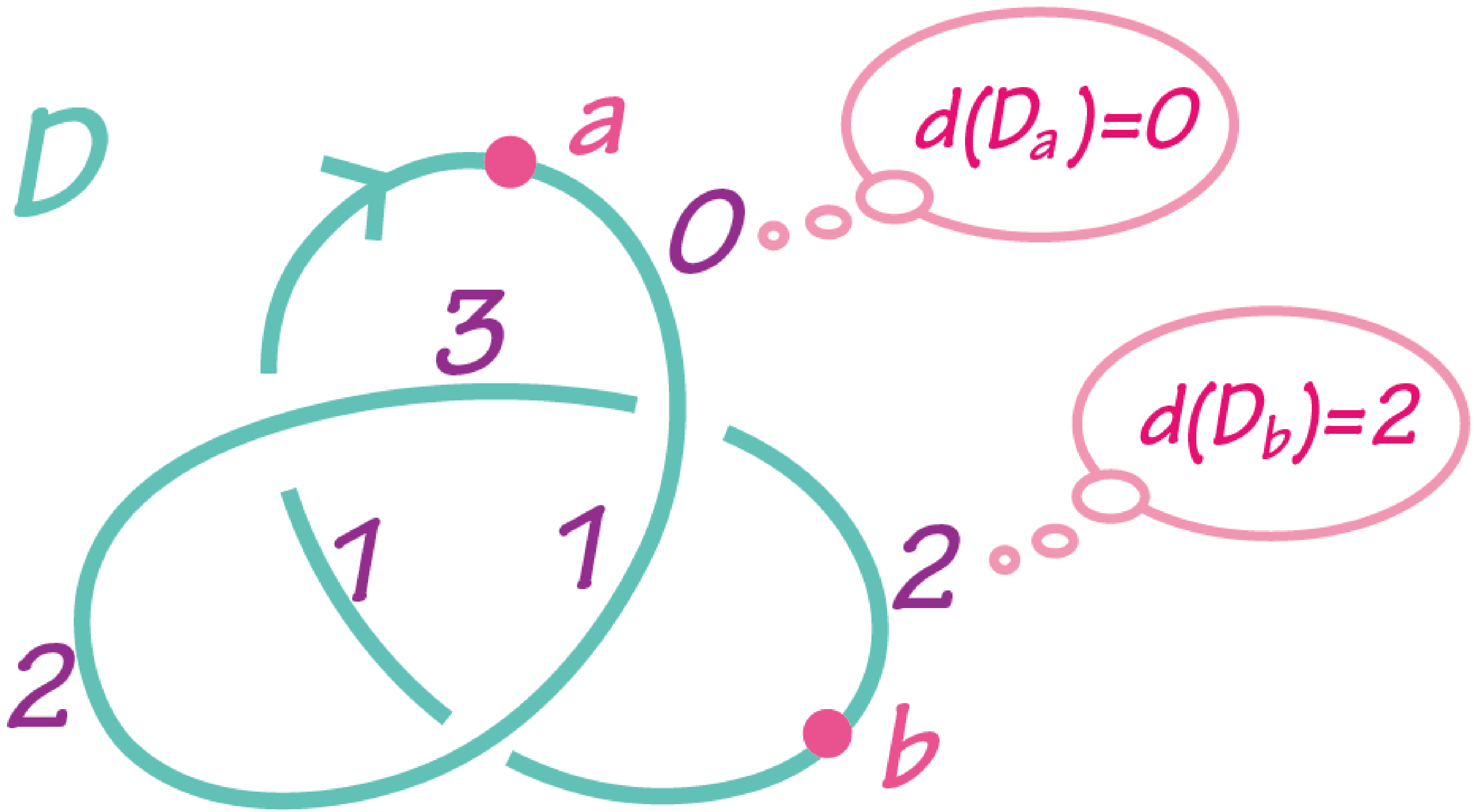


Remark

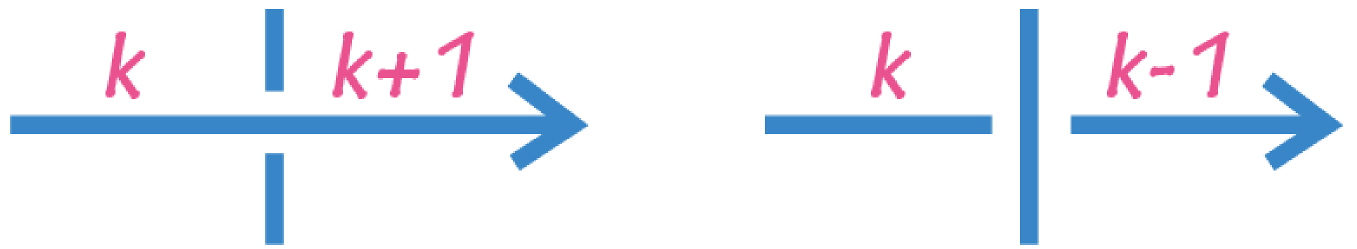
Warping degree depends on the choice of base point.

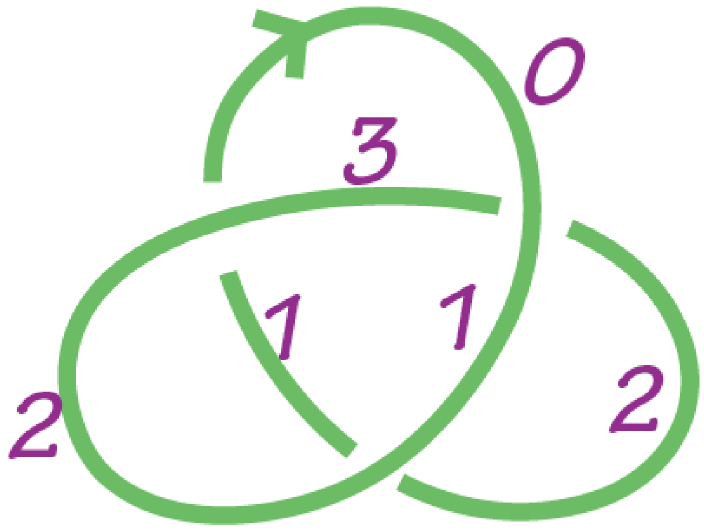


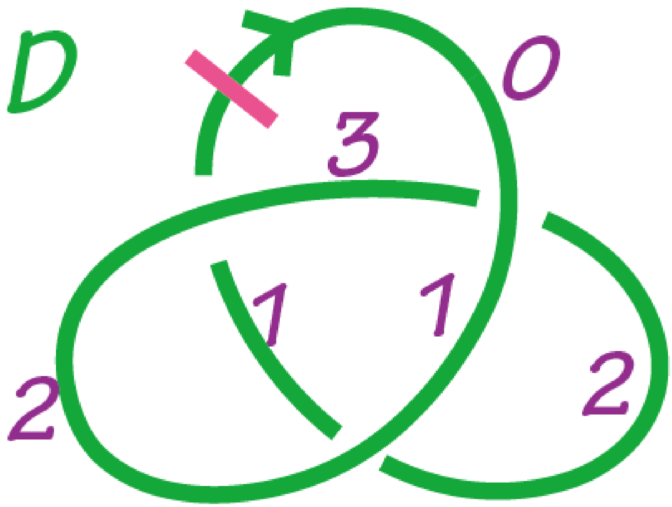
Warping degree labeling



Property



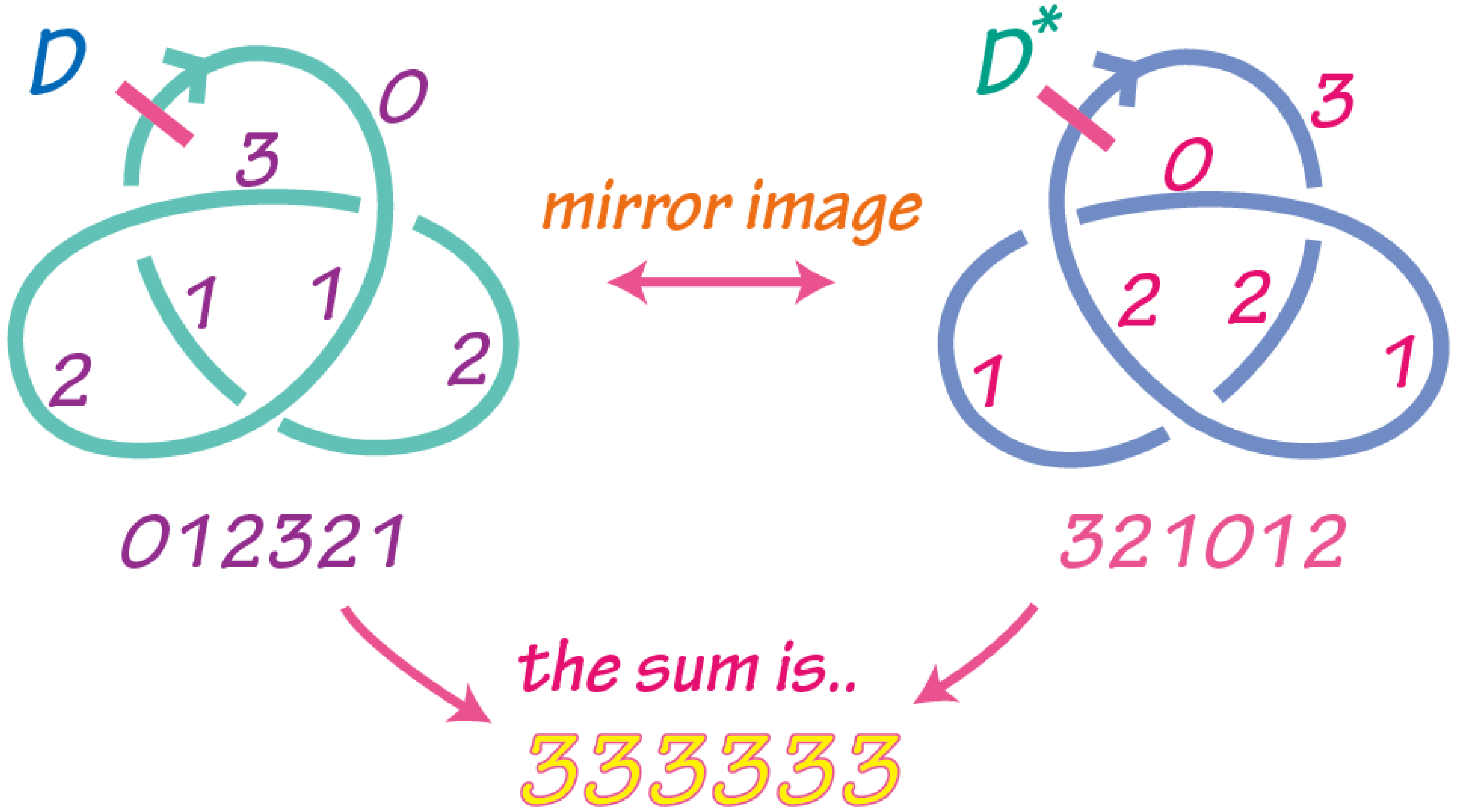




Warping degree sequence

012321 / cyclic permutation

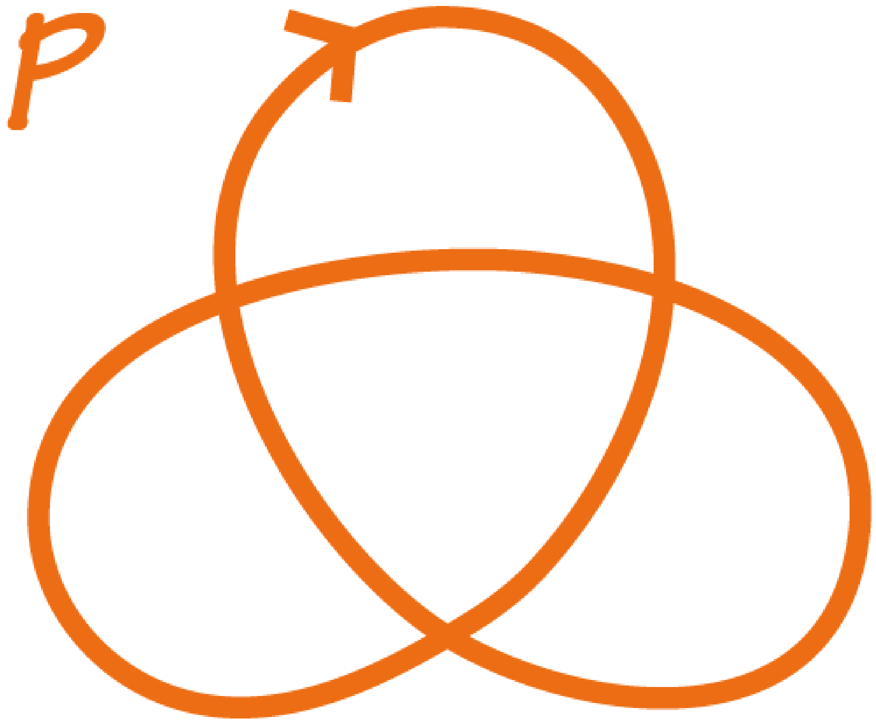
Property

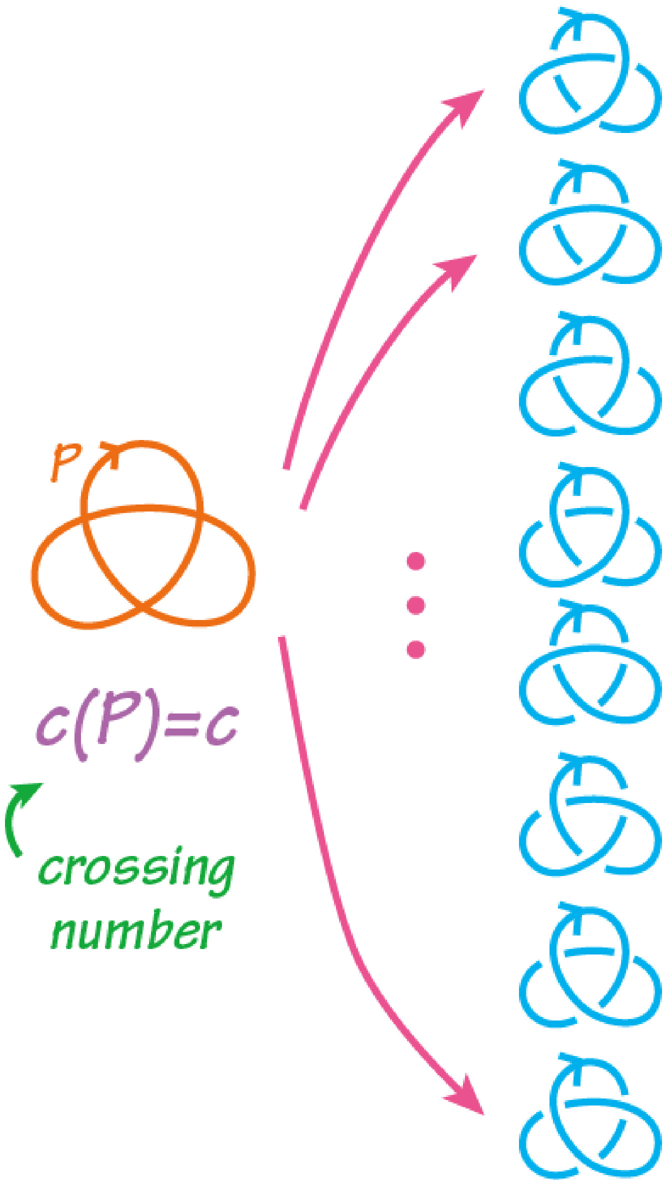


D
0
3
1
1
2
2
D*
3
0
2
2
1
1
mirror image
012321
321012
the sum is..
333333

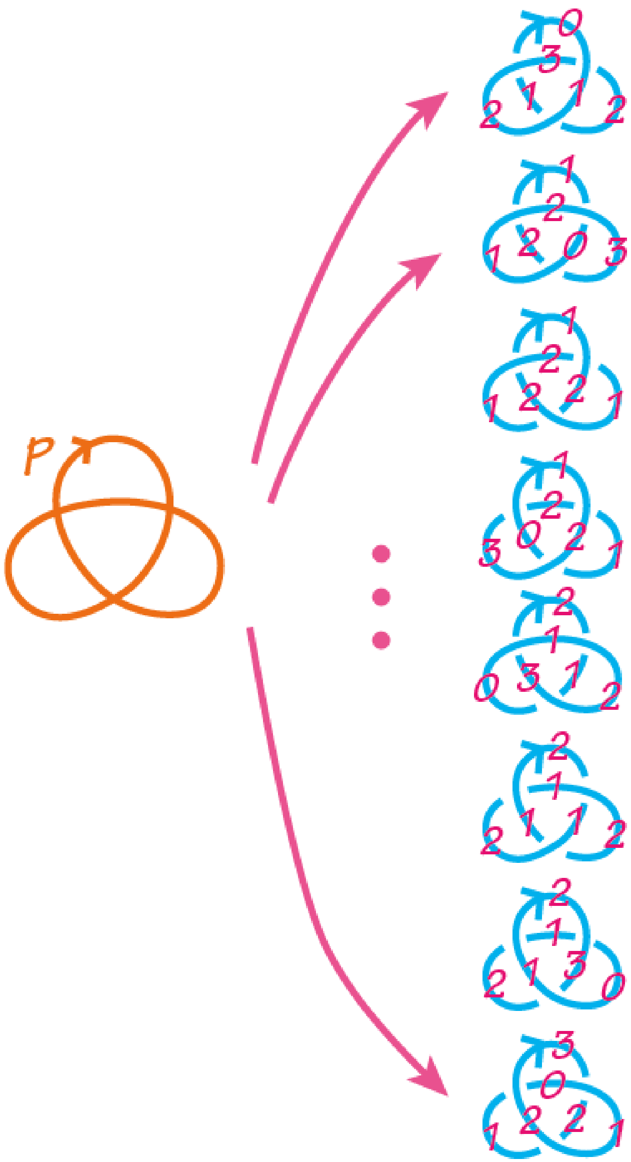
2. Warping matrix of a knot projection

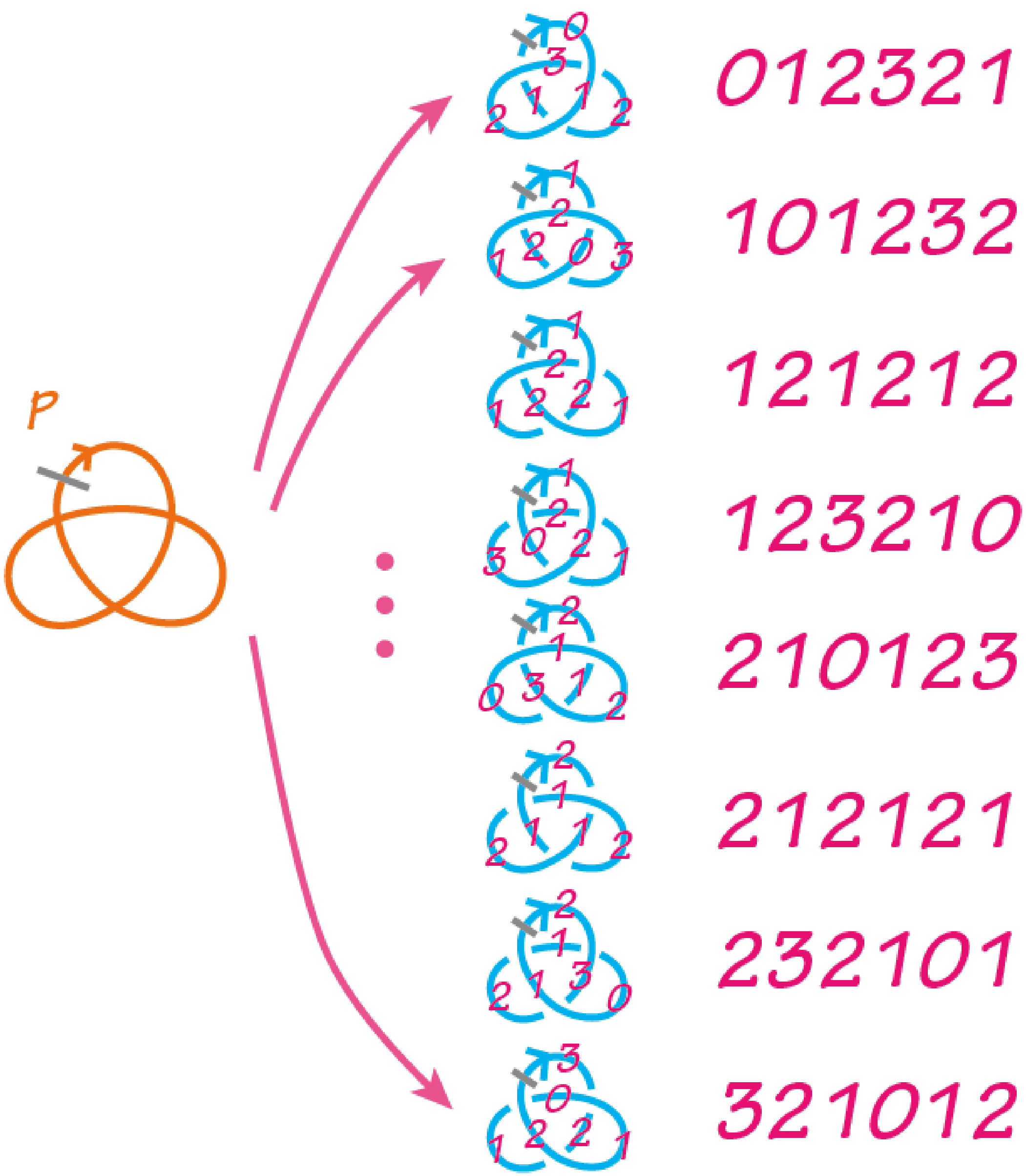
P: an oriented knot projection on S^2

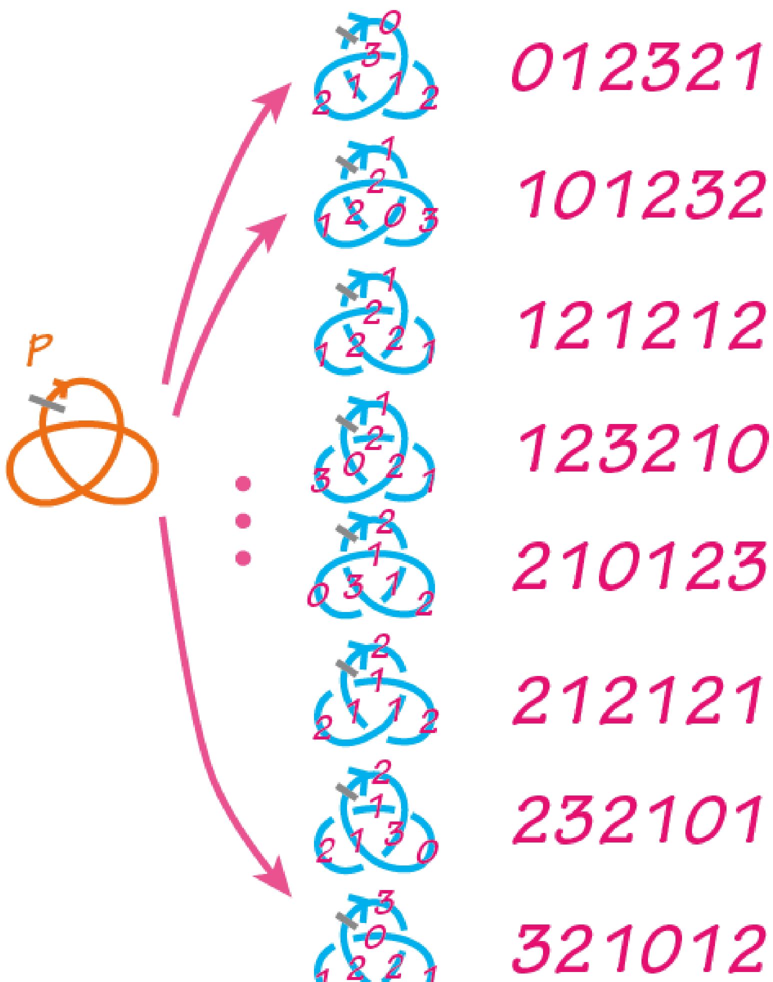




We have 2^c diagrams from P


p
0 3 2 1 1 2
1 2 1 2 0 3
1 2 1 2 2 1
1 2 3 0 2 1
2 1 0 3 1 2
2 1 2 1 1 2
2 1 2 1 3 0
3 0 1 2 2 1


p
012321
101232
121212
123210
210123
212121
232101
321012



$2^c \times 2c$ matrix

$$M(P)=\begin{pmatrix} 0&1&2&3&2&1\\ 1&0&1&2&3&2\\ 1&2&1&2&1&2\\ 1&2&3&2&1&0\\ 2&1&0&1&2&3\\ 2&1&2&1&2&1\\ 2&3&2&1&0&1\\ 3&2&1&0&1&2 \end{pmatrix}$$

Warping matrix of P

We consider warping matrices up to:

M()=

$$\begin{pmatrix} 0&1&2&3&2&1\\ 1&0&1&2&3&2\\ 1&2&1&2&1&2\\ 1&2&3&2&1&0\\ 2&1&0&1&2&3\\ 2&1&2&1&2&1\\ 2&3&2&1&0&1\\ 3&2&1&0&1&2 \end{pmatrix}$$

- Switching two rows.
- Applying a cyclic permutation on columns.

Property A $2^c \times 2c$ warping matrix $M(P)=(m_{ij})$ of a knot projection satisfies:

M()

$$=\begin{pmatrix} 0&1&2&3&2&1\\ 1&0&1&2&3&2\\ 1&2&1&2&1&2\\ 1&2&3&2&1&0\\ 2&1&0&1&2&3\\ 2&1&2&1&2&1\\ 2&3&2&1&0&1\\ 3&2&1&0&1&2 \end{pmatrix}$$

(1) $|m_{ij+1}-m_{ij}|=|m_{i1}-m_{i2c}|=1$ for $^{\forall}i, j$.

(2) At each column, n appears $\binom{c}{n}$ times.

(3) There are just 2^{c-1} disjoint pairs of rows s.t. the sum of the two rows is $(c\ c \ldots c)$.

(4) $^{\exists}(k\ \ k+1\ \ k \ldots k+1)$.

Example

M()

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

M()

$$\begin{pmatrix} 0&1&2&3&2&1 \\ 1&0&1&2&3&2 \\ 1&2&1&2&1&2 \\ 1&2&3&2&1&0 \\ 2&1&0&1&2&3 \\ 2&1&2&1&2&1 \\ 2&3&2&1&0&1 \\ 3&2&1&0&1&2 \end{pmatrix}$$

M() or M()

$$\begin{pmatrix} 0&1&2&1 \\ 1&0&1&0 \\ 1&2&1&2 \\ 2&1&0&1 \end{pmatrix}$$

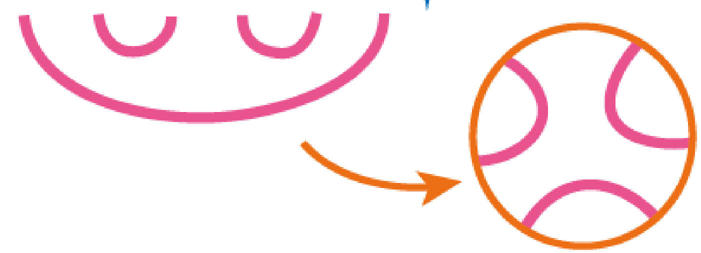
M()

$$\begin{pmatrix} 0&1&2&3&2&3&2&1 \\ 1&2&3&4&3&2&1&0 \\ 1&2&3&2&1&2&3&2 \\ 2&3&4&3&2&1&2&1 \\ 1&2&1&2&1&2&1&2 \\ 2&3&2&3&2&1&0&1 \\ 2&3&2&1&0&1&2&3 \\ 3&4&3&2&1&0&1&2 \\ 4&3&2&1&2&1&2&3 \\ 3&2&1&0&1&2&3&4 \\ 3&2&1&2&3&2&1&2 \\ 2&1&0&1&2&3&2&3 \\ 3&2&3&2&3&2&3&2 \\ 2&1&2&1&2&3&4&3 \\ 2&1&2&3&4&3&2&1 \\ 1&0&1&2&3&4&3&2 \end{pmatrix}$$

Lemma Each warping matrix represents the Gauss diagram of the knot projection.

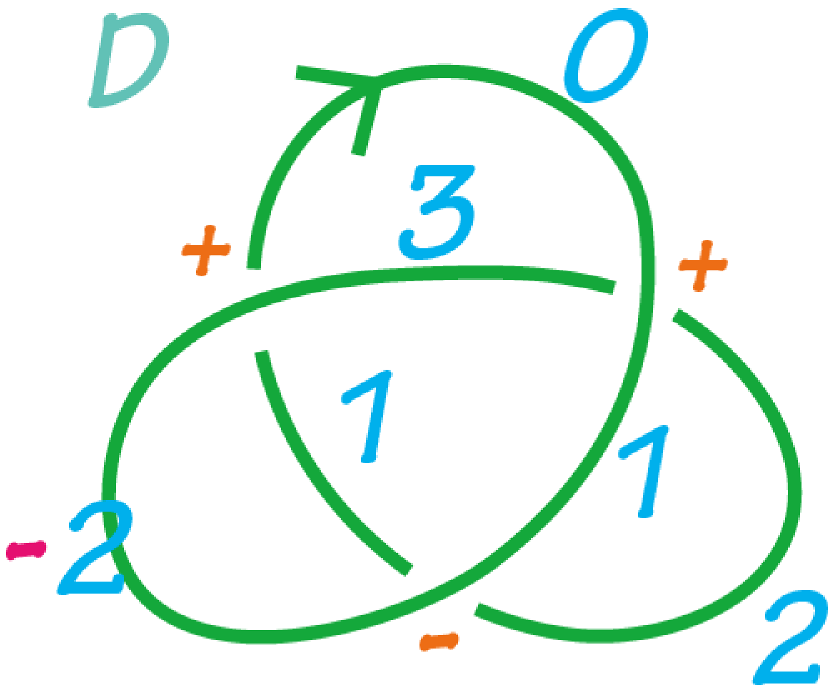
$$\begin{pmatrix} 0&1&2&1&2&1\\ 1&0&1&0&1&0\\ 1&2&1&2&3&2\\ 1&2&3&2&1&2\\ 2&1&0&1&2&1\\ 2&3&2&3&2&3\\ 2&1&2&1&0&1\\ 3&2&1&2&1&2 \end{pmatrix} \begin{pmatrix} -1&0&0&0&0&1\\ 1&-1&0&0&0&0\\ 0&1&-1&0&0&0\\ 0&0&1&-1&0&0\\ 0&0&0&1&-1&0\\ 0&0&0&0&1&-1 \end{pmatrix} = \begin{pmatrix} 1&1&-1&1&-1&-1\\ -1&1&-1&1&-1&1\\ 1&-1&1&1&-1&-1\\ 1&1&-1&-1&1&-1\\ -1&-1&1&1&-1&1\\ 1&-1&1&-1&1&-1\\ -1&1&-1&-1&1&1\\ -1&-1&1&-1&1&1 \end{pmatrix}$$

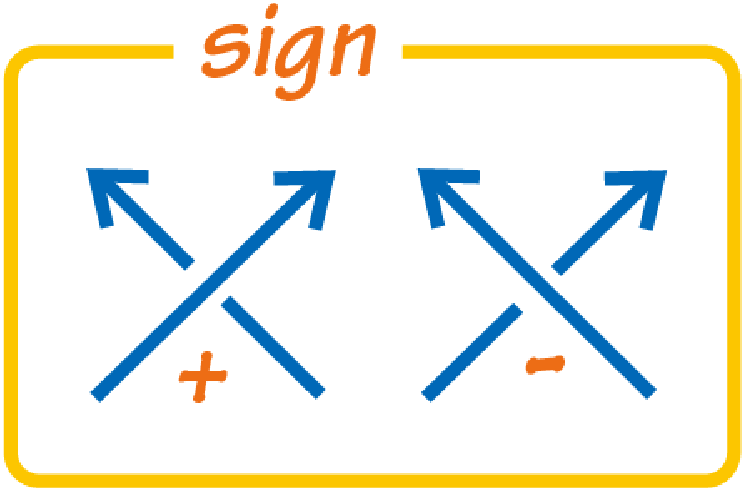
$=$ M()



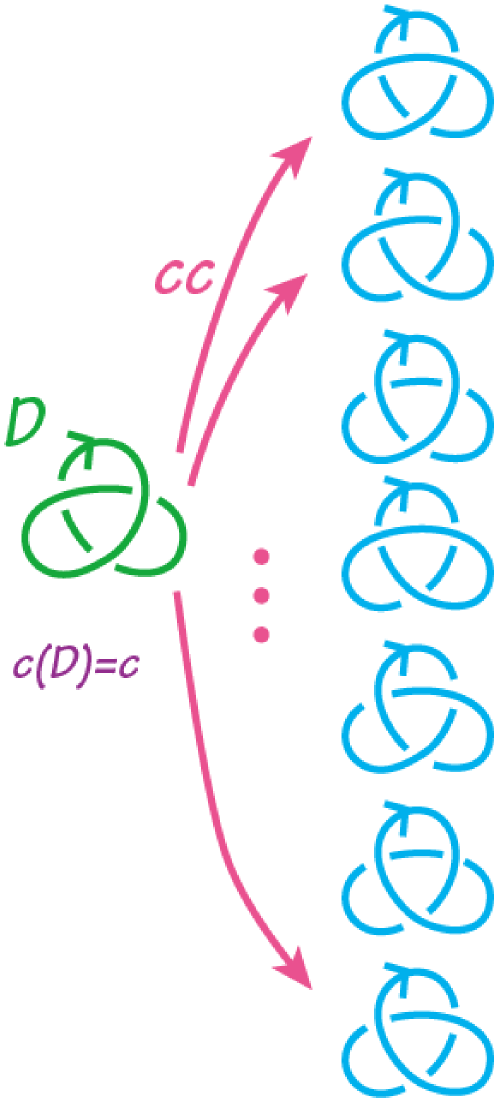
3. Warping matrix of a knot diagram

D: an oriented knot diagram on S^2

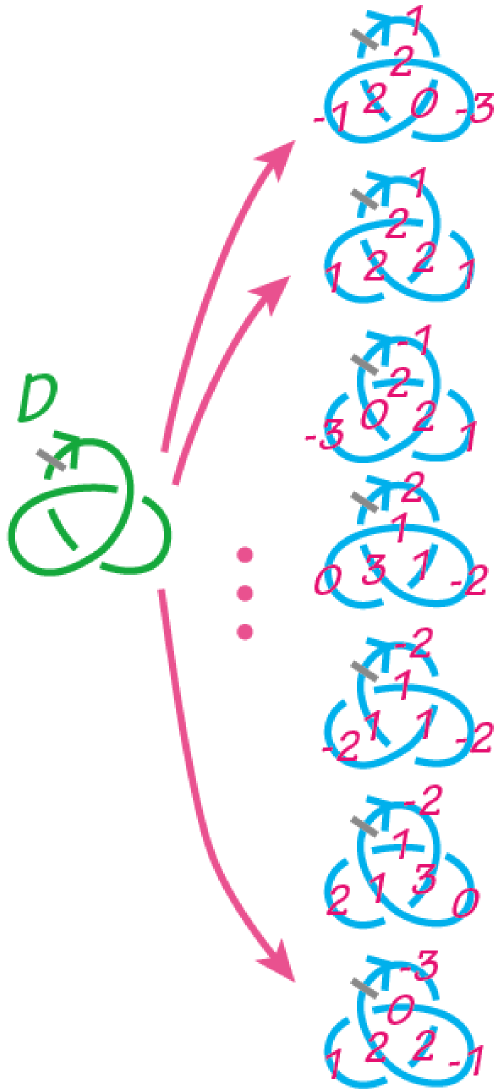


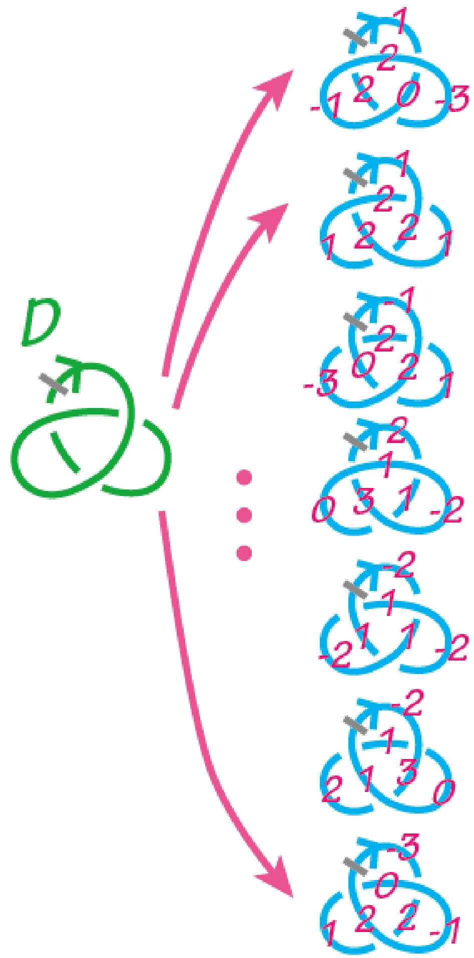


Signed warping degree labeling



We have $2^c - 1$ diagrams from D


D
1 2 -1 2 0 -3
1 2 1 2 2 1
-1 2 -3 0 2 1
2 1 0 3 1 -2
-2 1 -2 1 1 -2
-2 1 2 1 3 0
-3 0 1 2 2 -1



$10\bar{1}2\bar{3}2$

121212

$\bar{1}2\bar{3}210$

$21012\bar{3}$

$\bar{2}1\bar{2}1\bar{2}1$

$\bar{2}32101$

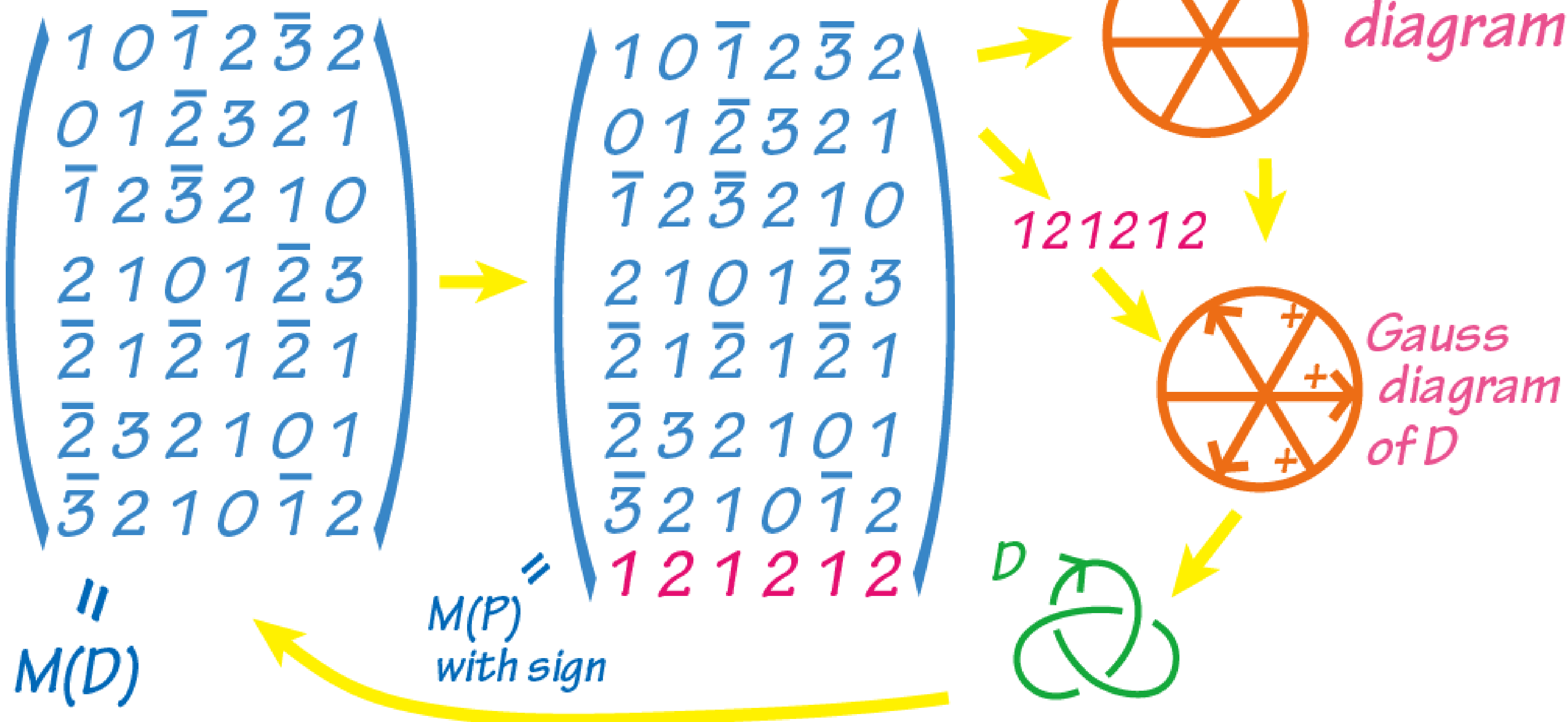
$\bar{3}210\bar{1}2$

$(2^c-1) \times 2c$ matrix

$$M(D)=\begin{pmatrix} 1 & 0 & \bar{1} & 2 & \bar{3} & 2 \\ 1 & 2 & 1 & 2 & 1 & 2 \\ \bar{1} & 2 & \bar{3} & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 & \bar{2} & 3 \\ \bar{2} & 1 & \bar{2} & 1 & \bar{2} & 1 \\ \bar{2} & 3 & 2 & 1 & 0 & 1 \\ \bar{3} & 2 & 1 & 0 & \bar{1} & 2 \end{pmatrix}$$

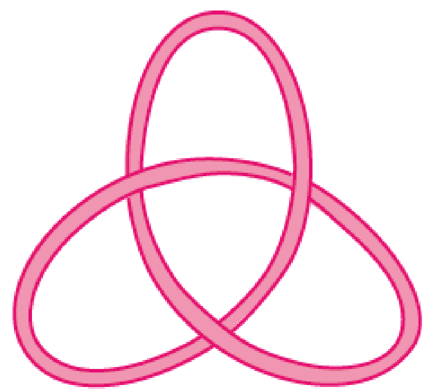
Warping matrix of D

Theorem Let D be an oriented knot diagram. Let M(D) be the warping matrix of D. We can reobtain D from M(D).



Corollary Each warping matrix represents an oriented knot.

$$\begin{pmatrix} 1 & 0 & \bar{1} & 2 & \bar{3} & 2 \\ 0 & 1 & \bar{2} & 3 & 2 & 1 \\ \bar{1} & 2 & \bar{3} & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 & \bar{2} & 3 \\ \bar{2} & 1 & \bar{2} & 1 & \bar{2} & 1 \\ \bar{2} & 3 & 2 & 1 & 0 & 1 \\ \bar{3} & 2 & 1 & 0 & \bar{1} & 2 \end{pmatrix} \longrightarrow$$



4. Puzzle

				3	
		3			
		2		0	
3			0		
	1		1		1
	2		2		2
0			3		
		0			

row

$$|m_{i\,j+1} - m_{i\,j}| = |m_{i\,1} - m_{i\,6}| = 1$$

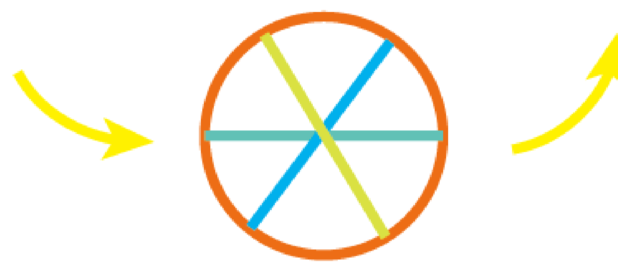
for $\forall i, j$.

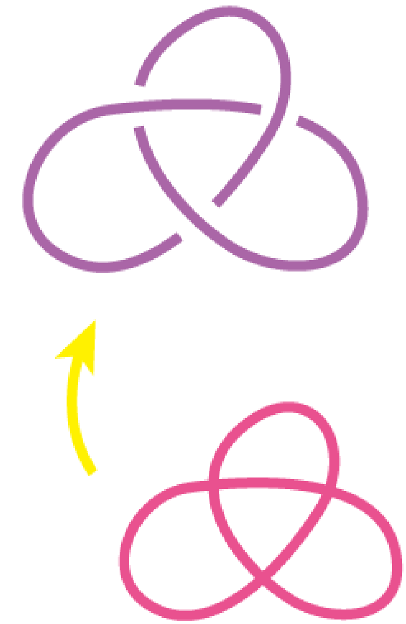
column

n appears $\binom{3}{n}$ times.

1	0	1	2	3	2
1	2	3	2	1	0
2	3	2	1	0	1
3	2	1	0	1	2
2	1	2	1	2	1
1	2	1	2	1	2
0	1	2	3	2	1
2	1	0	1	2	3

1 u	0 o	1 o	2 o	3 u	2 u
1 o	2 o	3 u	2 u	1 u	0 o
2 o	3 u	2 u	1 u	0 o	1 o
3 u	2 u	1 u	0 o	1 o	2 o
2 u	1 o	2 u	1 o	2 u	1 o
1 o	2 u	1 o	2 u	1 o	2 u
0 o	1 o	2 o	3 u	2 u	1 u
2 u	1 u	0 o	1 o	2 o	3 u





2					3		
	4				0		
1		3					
			0				
4				2			
1					4		
			1			4	
						1	
		0			3		3
1	2			1		1	
			3	2	3		
		3	2				
	3		3				
	3						3
		3					0
		4				2	

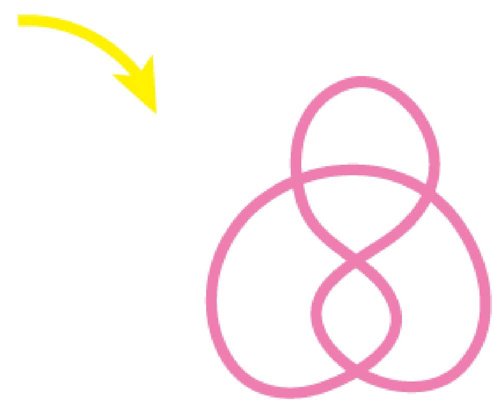
row

$|m_{i\,j+1}-m_{ij}|=|m_{i\,1}-m_{i\,8}|=1$

for $\forall i,j.$

column

n appears $\binom{4}{n}$ times.



Thank you for listening!